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THEORY AND DESIGN OF ELECTRICAL ROTATING MACHINERY.(U)  
APR 80 W J CARR N00014-72-C-0432

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THEORY AND DESIGN OF ELECTRICAL  
ROTATING MACHINERY

Final Report for Period  
Beginning May 1, 1972 and  
Ending October 31, 1979

Submitted to  
Office of Naval Research  
in April 1980

Principal Investigator:  
W. J. Carr, Jr.

Sponsored by:  
Power Program ONR Contract No. N00014-72-C-0432

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  The objective of this program was to contribute toward new and im- proved rotating machines for Naval applications, with emphasis on super- conducting machinery. Work has been performed on the theory of ac losses in multifilament superconductors and experiments were made to check the theory. A list of publications and abstracts of scientific papers published under the contract is given, and a review is given of the theory of losses. - 4		

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## 1.0 REPORTING PERIOD

This is the final technical report and covers the work started May 1, 1972 and completed October 31, 1979.

## 2.0 OBJECTIVES

### 2.1 General

The objectives of the program were to contribute toward new and improved rotating machines for Naval applications, particularly in the area of superconducting machinery.

### 2.2 Specific

A specific objective was to study the ac loss which occurs in multifilament superconductors, with the object of providing the information needed for the design of stable low-loss conductors for use in rotating machines.



### 3.0 INTRODUCTION

While this program was initially conceived as a program to study novel electrical machinery, and, in particular, to investigate the possibilities for the use of superconductors in electrical machines, it quickly became apparent that new concepts were largely limited by the ac losses which occur in superconductors. Consequently, after the first year the program evolved into a study of the losses in superconductors. Essentially all of the important results of this investigation have been published in readily accessible scientific journals, and therefore little purpose would be served in reproducing the bulk of this material in the final report. Only the abstracts of the published papers will be given here. Although a few of the papers listed in Section 6 were not published in scientific journals, the reason in all cases was the authors assessment that the results were not sufficiently important to warrant publication.

In addition to the abstracts of published papers, a review is given in this report of the theory of losses in multifilament superconductors. This review is taken from a chapter in a forthcoming book on the subject of ac loss, which accounts for the format.

Of the small amount of early work which was done directly on novel machine concepts the most significant is the design of an iron circuit for a homopolar motor.

#### 4.0 SUMMARY

Cont A macroscopic theory for superconductivity in multifilament superconductors was developed, and the theory was used to calculate the hysteresis and eddy current losses which occur in the presence of changing magnetic fields. Both the transverse field and the longitudinal field cases were considered, and also the self-field loss of an alternating transport current, along with some examples of the combined loss due to alternating applied field and transport current. The results are useful for the design of superconducting devices, such as superconducting motors and generators.

A small amount of additional work was done on studies of novel homo- and heteropolar motors.

## 5.0 PROGRAM PERSONNEL

The principal personnel who have contributed to this program are as follows:

- W. J. Carr, Jr. - Consultant in Magnetism and Superconductivity
- J. H. Murphy - Engineer, Cryogenics
- G. R. Wagner - Physicist, Cryogenics
- M. S. Walker - Now at Intermagnetics General Corporation (IGC).

Consultation has been provided by other members of the Machinery and Cryogenics Groups.

## 6.0 COMPLETE LIST OF PUBLICATIONS UNDER THE CONTRACT

1. W. J. Carr, Jr., "AC Loss in a Twisted Filamentary Superconducting Wire," Journal of Applied Physics, 45:929 (1974).

Abstract: Calculations are made for the ac losses in a twisted filamentary superconductor by assuming a continuum model with anisotropic conductivity. The Maxwell equations are solved for a long wire of radius  $R_0$  and twist length  $L$ . In terms of the classical skin depth  $\delta$ , appropriate for the conductivity perpendicular to the filaments, the losses are calculated for an applied magnetic field transverse to the wire axis in the case of (1) a uniform field ( $\sqrt{2} R_0/\delta \ll 1$ ,  $L/2\pi\delta \ll 1$ ), (2) shielding from surface currents ( $\sqrt{2} R_0/\delta \ll 1$ ,  $L/2\pi\delta \gg 1$ ), and (3) skin effect ( $\sqrt{2} R_0/\delta \gg 1$ ). Losses are also calculated for the case of a transport current in no applied field. The model can be used in more general cases and has the virtue of giving precise results with a minimum of auxiliary postulates.

2. W. J. Carr, Jr., "AC Loss in a Twisted Filamentary Superconducting Wire II," Journal of Applied Physics, 45:935 (1974).

Abstract: Hysteresis losses in a twisted filamentary superconducting wire are calculated for various cases corresponding to complete penetration of the magnetic field into the wire, shielding of the field by the supercurrents, and shielding due to eddy currents. In general, the wire may be divided into two regions: an outer saturated layer which tends to behave at the higher frequencies like a solid superconductor, and the interior where the average parallel electric field vanishes and the wire behaves like a collection of individual filaments.

3. J. H. Murphy, M. S. Walker, and W. J. Carr, Jr., "Theory of Alternating Field Losses in Cylindrical Twisted Multifilamentary Superconductors," Intermag Conf. Proc., IEEE Trans. Mag., 10:868 (1974).

Abstract: The theory of sinusoidal alternating field losses in a cylindrical multifilamentary superconducting wire is discussed. Hysteresis loss expressions are presented for the individual filament losses in the interior of the wire when these filaments are completely penetrated, and for the loss occurring in an outer current-saturated layer of the wire. The loss expressions presented were determined assuming a continuum model with anisotropic conductivity for the superconductor. The case of a permeability different from unity, applicable for weak fields, is discussed.

4. W. J. Carr, Jr., "Electromagnetic Theory for Filamentary Superconductors," Physical Review B, 11:1547 (1975).

Abstract: It is shown that a multifilament superconductor, made up of a bundle of twisted filaments embedded in a normal matrix, can be treated as a new state of matter with anisotropic electrical and magnetic properties. Macroscopic electromagnetic field vectors, which satisfy Maxwell's equations, are defined in terms of averages over the "microscopic" fields. However, the sources for the field, i.e., the current and charge densities and the magnetization and polarization, differ in some respects from those for ordinary matter. In particular, since the elementary magnetic dipole moments are distributed along lines rather than located at fixed points, the definition of the magnetization transverse to the filaments differs by a factor of 2 from that for ordinary matter, and the definition of the macroscopic current density is also slightly modified. Constitutive relationships among the field vectors in terms of permeabilities, dielectric constants, and conductivities are examined in the limits of strong and weak fields.

5. W. J. Carr, Jr., "Conductivity, Permeability and Dielectric Constant in a Multifilament Superconductor," *Journal of Applied Physics*, 46:4043 (1975).

Abstract: Results are given for the permeability, conductivity, and dielectric constant of a multifilament superconductor, treated as an anisotropic state of matter. The calculations are made for the limiting cases of strong and weak ac magnetic fields compared with the field required for complete penetration of the filaments. General expressions are given for the ac power loss.

6. W. J. Carr, Jr., M. S. Walker, and J. H. Murphy, "Alternating Field Loss in a Multifilament Superconducting Wire for Weak AC Fields Superposed on a Constant Bias," *Journal of Applied Physics*, 46:4048 (1975).

Abstract: Expressions are given for the alternating field loss in a twisted filamentary superconductor as a function of frequency and magnetic field, for the case of small ac applied fields which do not completely penetrate the filaments. The ac field is superimposed on a fixed bias field. In this limit the effective diamagnetism of the filament leads to a permeability less than unity for the composite. Expressions are presented for both the hysteresis and the eddy current loss.

7. M. S. Walker, W. J. Carr, Jr., and J. H. Murphy, "Loss Behavior in Twisted Filamentary Superconductors," *IEEE Trans. on Magnetics*, MAG-11, 1475 (1975).

Abstract: A careful comparison between theory and experiment is made for the transverse alternating field eddy current losses in three twisted mixed-matrix filamentary superconductors of the composition  $3\text{CuNi}:3\text{Cu}:1\text{NbTi}$ . The measurements were made over a wide frequency range, up to  $10^6$  Hz. Good agreement is obtained.

8. W. J. Carr, Jr., "Possibility for an All-Superconducting Synchronous Motor or Generator," Westinghouse Research Report 73-9J2-MACON-R1.

Abstract: Possibilities for a motor or generator with superconducting armature and field winding are discussed, and the ac losses that would be expected in the armature winding are calculated.

9. W. J. Carr, Jr., "Design Theory for a Fully Superconducting Synchronous Motor or Generator," Westinghouse Research Report 73-9J2-MACON-R2.

Abstract: Theory is developed for a superconducting synchronous motor or generator having both the field and the armature windings superconducting. An approximate design is given for a four-pole 30 MW operating at 180 rpm.

10. J. H. Murphy and W. J. Carr, Jr., "Eddy Current Losses in Twisted Multifilamentary Superconductors," Westinghouse Research Report 73-9J2-MACON-R3.

Abstract: Calculations are made for the eddy current losses in a multifilament superconductor having a copper sheath.

11. L. N. Wedman, "A Low Speed Direct Current Superconducting Heteropolar Motor," Report EM4562.

Abstract: Important progress in assessing the limits of ratings for new concept electrical machines for ship propulsion applications has been achieved. The superconducting heteropolar motor has been identified as the most promising machine concept for slow-speed, high-power use. Maximum utilization of the drum homopolar machine is limited by transmission line current ratings, and may be greatly improved if an in-line drive is required, thus allowing the use of the homopolar torque converter. The magnetic disc homopolar motor with water cooling appears to be the preferred concept for low-speed homopolar motor applications. Technical progress in the conceptual configuration of the superconducting

heteropolar machine is presented, together with a report on progress on the solution of commutation problems in such machines.

12. W. J. Carr, Jr., M. S. Walker, D. W. Deis, and J. H. Murphy, "Hysteresis Loss in a Multifilament Superconductor," *Advances in Cryogenic Engineering*, 22, 428 (1977).

Abstract: Theory of hysteresis loss in a multifilament superconductor is reviewed and measurements are presented for a mixed-matrix NbTi conductor. The measurements were made calorimetrically as a function of frequency and transverse ac magnetic field, for a 50 kOe dc bias field, and also with no bias. Both a linear ac field dependence and a cubic field dependence were observed, corresponding to the case of complete penetration of the filaments, and partial penetration. Comparison of experiment with calculation was made by evaluating the critical current density from the measurements and the loss expression, and comparing this value with a direct measurement of  $j_c$  from transport current. Good agreement was obtained.

13. W. J. Carr, Jr., "Parallel Field Losses in Twisted Multifilament Superconductors," *Sixth Symposium on Engineering Problems of Fusion Research*, IEEE Pub. No. 75CH1097-5-NPS (1975), p. 152.

Abstract: In some applications of multifilament superconducting wire an appreciable component of a time dependent magnetic field exists along the twisted filaments, and across the matrix, and loss and stability problems are introduced. It has been suggested by various authors that these effects can be minimized by periodically introducing reverse twist in the wire. A calculation is given for the eddy current loss that can be expected for such a conductor. The calculation also applies for the case where the twist is uniform but the direction of the magnetic field reverses along the length of the conductor.



14. W. J. Carr, Jr., "Longitudinal and Transverse Field Losses in Multifilament Superconductors," IEEE Transactions on Magnetics, Vol. MAG-13, 192 (1977).

Abstract: A review is given of the method of calculating losses in a composite twisted multifilament superconductor based on the anisotropic continuum model, and a summary of results are shown for the loss as a function of frequency due to longitudinal and transverse applied fields. In both cases three frequency ranges may be distinguished in which (1) the magnetic field in the superconductor is just the applied field, (2) the field produced by internal currents becomes important, and (3) skin effect develops in the eddy currents, which flow transverse to the filaments. In addition to these losses in the body of the superconductor (characterized by a vanishing component of electric field parallel to the filament axes), the losses in the current saturated boundary layer is described. Some discussion is also given on the use of ac field losses in calculating the "ramped" field case.

15. W. J. Carr, Jr., "Longitudinal Field Losses in Multifilament Superconductors over a Range of Frequencies," J. Appl. Phys., 2022 (1977).

Abstract: The power loss in a twisted multifilament superconductor due to a time-dependent longitudinal magnetic field is calculated as a function of frequency. The longitudinal field is assumed to have a sinusoidal dependence on both space and time, alternating changing direction along the conductor over a length  $\ell$ . The eddy current loss, as a function of frequency, initially is approximately proportional to  $\ell^2$  and rises as the square of the frequency, until it reaches a plateau which tends to be frequency independent and inversely proportional to  $\ell^2$ . After a further increase with frequency, the skin effect region is reached with frequency to the one-half power dependence. The current-density distribution in the conductor is calculated, and it is shown that for large  $\ell$  the entire cross section of the conductor can become current saturated if the amplitude of the ac field is greater than  $L\lambda j_c$  where  $L$  is the twist length,  $j_c$  is the critical current density, and  $\lambda$  is the fraction of superconductor.

16. W. J. Carr, Jr., J. H. Murphy, and G. R. Wagner, "Alternating Field Losses in Filamentary Superconductor Carrying DC Transport Current," Advances in Cryogenic Engineering, Volume 24, 415 (1978).

Abstract: Alternating field losses in multifilament superconductors have been calculated previously only for special limiting cases, i.e., for the case of full field penetration of the filaments, or for the case of no transport current. From these limiting values interpolation formulas are given here for the full range of alternating field and dc transport current, assuming only that the frequency is below the critical frequencies for internal field effects, and that saturation effects in the outer filaments may be neglected. The results are compared with some measurements.

17. W. J. Carr, Jr., "Magnetic Circuit Design for a Homopolar Motor," Westinghouse Research Report 76-9C30-MACON-P2. Presented to the Winter Meeting IEEE Power Engineering Society in New York, January 1978.

Abstract: The use of iron to enhance the flux density in a homopolar machine is limited by the presence of a large circumferential field which tends to saturate the iron in the wrong direction. It is shown that the use of an iron structure with optimized circumferential gaps can overcome this difficulty, and in many cases eliminate the necessity of a compensating current.

18. W. J. Carr, Jr., Ben Clawson, and Wayne Vogen, "AC Losses in Superconducting Solenoids," IEEE Transactions on Magnetics, Vol. MAG-14, 617 (1978).

Abstract: Losses were measured in a variety of different solenoids made from various types of superconductors. Both single filament and multifilament, twisted and untwisted, conductors were used. The results were compared with calculated losses for these cases.

19. W. J. Carr, Jr., "AC Loss from the Combined Action of Transport Current and Applied Field," To appear in the Proceedings of the 1978 Applied Superconductivity Conference, 1978.

Abstract: The ac loss in a superconductor resulting from the combined action of an alternating transport current and in-phase ac transverse field is calculated for a slab, and normalized in a form which can be applied approximately to a circular wire. Some results for limiting cases are also directly calculated for a wire.

## 7.0 THEORY OF FILAMENTARY SUPERCONDUCTORS

### 7.1 Introduction

A composite conductor consists of superconducting filaments embedded in a normal metal matrix where, typically, the filament diameter is in the range of 1 to 100 microns. The filaments are usually twisted about the axis of the wire. Superconductors of this type were originally developed to provide stability against flux jumps, but in most cases they offer the additional advantage of a low-loss conductor for changing magnetic fields. From the loss standpoint an ideal composite for some applications would consist of very fine filaments fully transposed in an insulating matrix. But stability requirements of heat transfer and the availability of current sharing paths dictate the use of a metallic matrix, and, further, the only transposition which can easily be introduced is that of twist. The existence of the metallic matrix has the disadvantage that induced currents can circulate and produce a joule heating in the matrix. On a large macroscopic scale this loss is simply an eddy current loss similar to that which occurs in a normal material. The eddy currents tend to flow transverse to the filament axes, and on a microscopic scale they may either pass through the superconducting filaments or flow around the filaments depending on conditions at the filament-matrix interface. At the surface of a conductor in a transverse field the component of eddy current density flowing normal to the surface tends

to enter the surface filaments and flow along the surface, since it offers a low resistance return path. The number of layers of filaments near the surface involved in the return path depends on the number of filaments required to carry the current.

Some of the earliest theoretical work on losses in twisted composites was done by a group at the Rutherford Laboratory,<sup>1</sup> and by Morgan<sup>2</sup> and his associates at Brookhaven. Analysis was based on the idea of leakage ("coupling") currents flowing between circuits formed by the filaments. This approach, which has been extended by many authors, was highly fruitful in providing initial insight and understanding, but for some problems quantitative results from the circuit concept are difficult to obtain. The analysis presented here will be based on field theory.

## 7.2 The Anisotropic Continuum Model

Losses in a composite superconductor consist of an eddy current loss due to the matrix and a hysteresis loss due to the filaments. To obtain the former an average electric field in the matrix is required, while the hysteresis can be calculated with the use of standard expressions when the magnetic field acting on the filaments and the current flowing in them is known. (For example, after the field and current are obtained one may revert back to the filament picture to compute the hysteresis.) Both the electric field and the

- 
1. M. N. Wilson, C. R. Walters, J. D. Lewin and P. F. Smith, J. Phys. D: Appl. Phys. 3, 1518 (1970).
  2. G. H. Morgan, J. Appl. Phys. 41, 3673 (1970).

magnetic field needed for the calculation may be obtained by solving a set of Maxwell equations for the composite treated as a smeared out continuum.<sup>3</sup> The continuum is like a fictitious state of matter having highly anisotropic properties. The conductivity and permeability in a direction parallel to the filament axes at any point differ from those perpendicular to the filaments. The Maxwell equations for the continuum are obtained in the same manner as they are derived for a pure superconductor, i.e., as a volume average over a set of Maxwell-Lorentz equations, except that now the volume element is larger, so that the average is over filament and matrix.<sup>4</sup> Since differentiation commutes with the average operation the form of the equations is unaffected by the average, and the operation may be performed either in one step, or as two steps where equations on a smaller macroscopic scale are first derived and used as microscopic equations for the next average.

Consider the averaged Maxwell-Lorentz equations given by

$$\text{curl } \langle \underline{e} \rangle = - \frac{\partial}{\partial t} \langle \underline{b} \rangle \quad (7-1)$$

$$\text{curl } \langle \underline{b} \rangle \approx 4\pi \langle \underline{j} \rangle \quad (7-2)$$

---

3. W. J. Carr, Jr., J. Appl. Phys. 45, 929 (1974).

4. One obtains a family of Maxwell equations, starting with a macroscopic scale large compared with atomic dimensions, followed by a scale large compared with vortex structure, followed, in turn, by a scale large compared with filament size.

$$\text{div } \langle \mathbf{e} \rangle = 4\pi c^2 \langle \rho_{\text{mic}} \rangle \quad (7-3)$$

$$\text{div } \langle \mathbf{b} \rangle = 0 \quad (7-4)$$

and assume that to good approximation one can recognize two kinds of currents:

$$\mathbf{i} = \mathbf{i}_{\text{mag}} + \mathbf{i}_{\text{free}} \quad (7-5)$$

where  $\mathbf{i}_{\text{mag}}$  circulates locally, while  $\mathbf{i}_{\text{free}}$  circulates over dimensions large compared with the filament diameter. Then the magnetization is given by

$$\text{curl } \mathbf{M} = \langle \mathbf{i}_{\text{mag}} \rangle \quad (7-6)$$

and the Maxwell current density is given by

$$\mathbf{j} = \langle \mathbf{i}_{\text{free}} \rangle \quad (7-7)$$

Similarly  $\rho_{\text{mic}}$  has a "free" and a "bound" part leading to

$$\langle \rho_{\text{mic}} \rangle = \rho - \text{div } \mathbf{P} \quad (7-8)$$

where  $\rho$  is the Maxwell charge density and  $\underline{P}$  the polarization. By defining  $\underline{B}$  and  $\underline{E}$  in the usual way and by taking

$$\underline{H} = \underline{B} - 4\pi \underline{M} \quad (7-9)$$

$$\underline{D} = \frac{\underline{E}}{c^2} + 4\pi \underline{P} \quad (7-10)$$

one obtains the Maxwell equations

$$\text{curl } \underline{E} = - \dot{\underline{B}} \quad (7-11)$$

$$\text{curl } \underline{H} = 4\pi \underline{j} \quad (7-12)$$

$$\text{div } \underline{D} = 4\pi \rho \quad (7-13)$$

$$\text{div } \underline{B} = 0. \quad (7-14)$$

Although the same symbols apply here as in the case of a pure (non-composite) bulk superconductor, the meaning of the vectors is not the same, and the difference is simply the difference of scale. Relative to the macroscopic scale considered here the free current density is essentially that which comes from the local net transport current in individual filaments, along with that which flows as eddy current across the matrix (Figure 7-1). A small amount of transport current can flow in the



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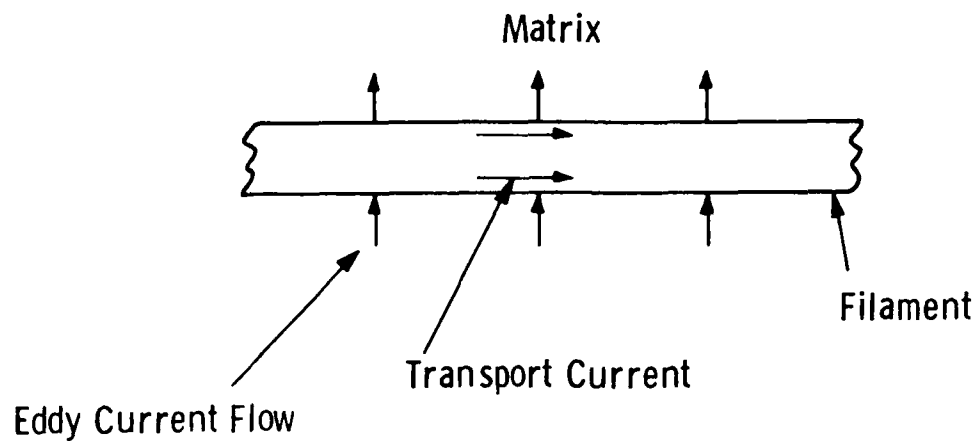


Fig. 7. 1 — Transport current along the filament and eddy current flow transverse to the filament. The average over filament and matrix gives the current density  $j$ .

matrix but this is usually negligible. In general  $i_{\text{mag}}$  can include:  
 (1) atomic currents, but these will be neglected,<sup>5</sup> (2) the vortex currents, which are also neglected, and (3) the surface and bulk shielding current confined to a given superconducting filament (Figure 7-2). With neglect of the first two sources it can be shown that<sup>6,7</sup>

$$\vec{M}_{||} = \frac{\lambda}{2a} \int \vec{R} \times \vec{i}_{\perp} da \quad (7-15)$$

and

$$\vec{M}_{\perp} = \frac{\lambda}{a} \int \vec{R} \times \vec{i}_{||} da \quad (7-16)$$

where  $\vec{M}_{||}$  and  $\vec{M}_{\perp}$  represent the magnetization parallel and transverse to the filament axes,  $\vec{i}_{||}$  and  $\vec{i}_{\perp}$  are the parallel and perpendicular

---

5. This assumption must be reexamined for a matrix having ferromagnetic elements which lead to ferromagnetism or strong paramagnetism.

6. W. J. Carr, Jr., Phys. Rev. B 11, 1547 (1975).

7. W. J. Carr, Jr., J. Appl. Phys. 46, 4043 (1975).

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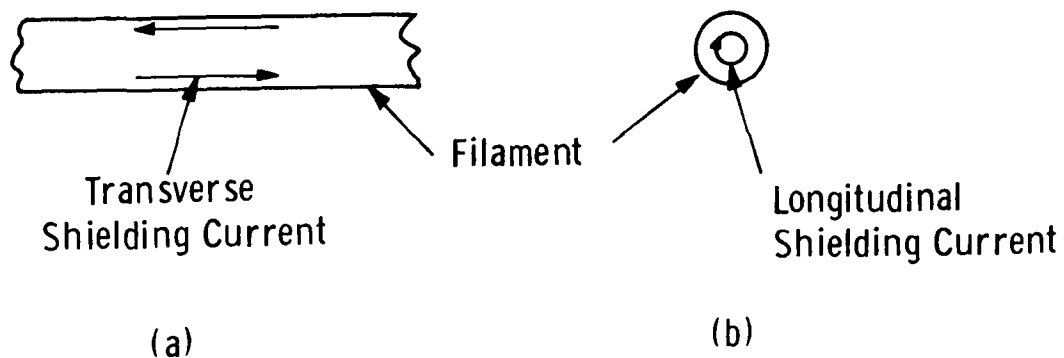


Fig. 7. 2 — Currents in the filament tending (a) to shield the transverse and (b) the longitudinal magnetic fields. These currents do not contribute to the macroscopic  $j$  but they give rise to the magnetization.

shielding current densities flowing in a filament,<sup>8</sup>  $\lambda$  is the fraction of superconductor area in the composite,  $\underline{R}$  is a radial vector from the axis of the filament, and  $a$  is the area of the filament. The polarization  $\underline{P}$  may be calculated in a similar way.<sup>6,7</sup>

The continuum model obviously applies best for the case of a composite having a large number of fine filaments. A minor disadvantage of the model is that calculated distances such as skin depths have no particular significance when they become smaller than the diameter of a filament.

### 7.3 Symmetry of the Continuum

The symmetry of the continuum is helical, as determined by the twisted filaments. Consider the path of a filament in terms of the cylindrical coordinates  $R, \theta, z$  of the conductor, which is assumed to have a circular cross section. If  $ds$  is an element of distance along the path then

$$\begin{aligned} ds &= dz \hat{a}_z + R d\theta \hat{a}_\theta \\ &= dz \left( \hat{a}_z + R \frac{d\theta}{dz} \hat{a}_\theta \right) \end{aligned} \quad (7-17)$$

- 
8. Since the transport current tends to give no contribution in these expressions  $i_{||}$  may be taken approximately as the total of the bulk and surface current density flowing along the axis of the filament.  $i_{\perp}$  tends to be induced by longitudinal fields and circulates around the filament cross section.

where  $\hat{a}_z$  is a unit vector in the  $z$  direction along the axis of the conductor, and  $\hat{a}_\theta$  is a unit vector in the direction around the circumference. For a uniform twist of  $2\pi$  radians in a length  $L$  centimeters along the axis

$$d\vec{s} = dz \left( \hat{a}_z + \frac{2\pi R}{L} \hat{a}_\theta \right) \quad (7-18)$$

and the unit vector along the filament axis is given by

$$\frac{d\vec{s}}{ds} = \frac{\hat{a}_z + \frac{2\pi R}{L} \hat{a}_\theta}{\left[ 1 + \left( \frac{2\pi R}{L} \right)^2 \right]^{1/2}} \quad (7-19)$$

The unit vector in the radial direction  $\hat{a}_R$  is orthogonal to  $d\vec{s}/ds$ , and a third orthogonal unit vector may be formed from the vector product of these two. If the unit vectors relative to the filament axes are labeled  $\hat{a}_1$ ,  $\hat{a}_2$  and  $\hat{a}_3$  then (Figure 7-3)

$$\hat{a}_1 = \frac{\hat{a}_z + \left( \frac{2\pi R}{L} \right) \hat{a}_\theta}{\left[ 1 + \left( \frac{2\pi R}{L} \right)^2 \right]^{1/2}} = \hat{a}_{||} \quad (7-20)$$

$$\hat{a}_2 = \hat{a}_R \quad (7-21)$$

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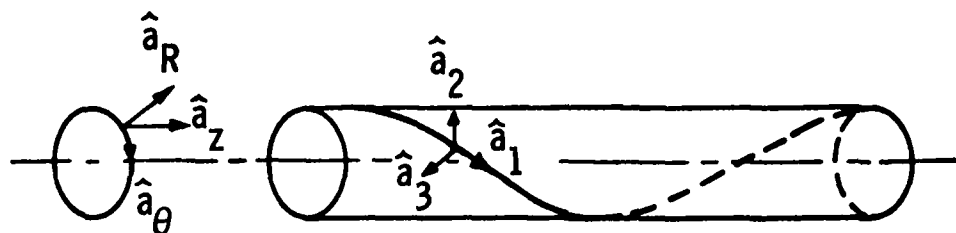


Fig. 7. 3 — Unit vectors  $\hat{a}_1$ ,  $\hat{a}_2$ , and  $\hat{a}_3$  for the twisted filaments and the unit vectors  $\hat{a}_R$ ,  $\hat{a}_\theta$ ,  $\hat{a}_z$  for the cylindrical conductor.

$$\hat{a}_3 = \frac{\hat{a}_\theta - \left(\frac{2\pi R}{L}\right) \hat{a}_z}{\left[1 + \left(\frac{2\pi R}{L}\right)^2\right]^{1/2}} \quad (7-22)$$

where  $\hat{a}_3$  lies on a cylindrical surface. Anisotropy in properties must be referred to the axes 1, 2, 3. In general, for simplicity, properties along the 2 and 3 axes are assumed to be the same.

#### 7.4 Constitutive Equations

Maxwell equations for matter require three constitutive equations for their solution, and for the present equations two of these may be taken as

$$\vec{B} = \mu_{\perp} \vec{H}_{\perp} + \mu_{||} \vec{H}_{||} \quad (7-23)$$

$$\vec{D} = \frac{1}{c} \left( K_{\perp} \vec{E}_{\perp} + K_{||} \vec{E}_{||} \right) \quad (7-24)$$

where  $\mu$  is the permeability and  $K$  a dielectric constant, and, again, the subscripts  $||$  and  $\perp$  refer to the filament paths. ( $\vec{H}_{||}$  is equal to  $H_1 \hat{a}_1$  while  $\vec{H}_{\perp}$  indicates  $H_2 \hat{a}_2 + H_3 \hat{a}_3$ .) For the magnetic problems of interest here, (7-24) is of minor importance.

If  $\sigma$  represents conductivity, the third constitutive equation can be written by considering two cases:

For  $j_{||} \leq \lambda j_c$

$$E_{||} \approx 0 \quad (7-25)$$

and

$$j_{\perp} = \sigma_{\perp} E_{\perp} \quad (7-26)$$

and the Maxwell equations will determine the particular value of  $j_{||}$ .

For  $j_{||}$  slightly greater than  $\lambda j_c$  the constitutive equation may be approximated by

$$\begin{aligned} j_{\perp} &\approx \sigma_{\perp} E_{\perp} + \sigma_{||}' E_{||} + \lambda j_c \frac{E_{||}}{|E_{||}|} \\ &\approx \sigma_{\perp} E_{\perp} + \lambda j_c \frac{E_{||}}{|E_{||}|} \end{aligned} \quad (7-27)$$

The meaning of these relations is as follows: When the filaments in the neighborhood of a point each carry their critical current, the current density parallel to the filaments is  $\lambda j_c$  when averaged over filament and matrix. At this value of current density the filaments are "saturated," and an appreciable electric field  $E_{||}$  is required to produce a small increase in current density  $\sigma_{||}' E_{||}$ , which can usually be neglected. Conversely, if  $j_{||}$  is just at or below  $\lambda j_c$  the electric field  $E_{||}$  is quite small, and an appreciable electric field can exist only in a direction



perpendicular to the filaments. In the latter case it is not to be supposed that no parallel microscopic electric field exists, since this would imply no hysteresis loss. Nor is it supposed that the average value  $E_{||}$  is precisely zero ( $E_{||}$  is zero only if there is no transport current in the filament). However for unsaturated transport in a filament,  $E_{||}$  is small and may be set equal to zero for the purpose of calculating the eddy current loss and magnetic field.

### 7.5 The Applied Field

In the analysis of a composite superconductor it is important to distinguish between various magnetic fields. Let the applied field  $H_A$  always denote the field which is applied to the composite conductor, assumed to be uniform over the conductor cross section. If the conductor represents a section of one turn in a coil, or one strand in a cable, then  $H_A$  is the field which would exist in the cavity left when a section of the conductor is imagined to be removed. For a conductor in a coil,  $H_A$  results from currents in all the other turns plus any field due to diamagnetic effects of the other turns. Even the copper sheath, which generally exists around the filamentary part of the conductor can influence the applied field if the field is very rapidly changing. For a conductor in a coil a formal expression for the applied field is given by

$$H_A = H_{\text{ext. sources}} + \int \frac{j(r')(r - r')}{|r - r'|^3} dv' \quad (7-28)$$

where the integral is over the total volume of the coil, except for the small part occupied by the section of conductor under consideration. If the currents are entirely within the conductors

$$H_A = H_{\text{ext. sources}} + \int \frac{j(r')(r - r') dv'}{|r - r'|^3} - \int \frac{(r - r') \text{div}' M(r') dv'}{|r - r'|^3} + \int \frac{(r - r')}{|r - r'|^3} M(r') \cdot dS' \quad (7-29)$$

where the integrals are over the volume and surfaces of all conductors, with only the region near the point of evaluation excluded. The method used to approximate these integrals will depend upon whether the coil is tightly wound or loosely wound. In the former case one can smear out the current density to define a current density for the coil, along with a magnetization for the coil, and make use of demagnetizing factors and the field distribution in standard solenoids. For a loosely wound coil the coil magnetization can be neglected, but only the distant conductors can be smeared out. Although the applied field may be very difficult to obtain, it is a problem regarded here as separate from the loss problem itself.

#### 7.6 The Local Field Applied to a Filament

Since hysteresis loss expressions are given in terms of the magnetic field which is applied to a superconductor, the field of interest

for hysteresis calculations in a composite conductor is the field "applied" to a superconducting filament. This field will be called the local field, which may be computed by examining the field that would exist in a cylindrical cavity resulting from an imaginary removal of a section of filament. Consequently<sup>9</sup>

$$H_{loc} = H + 2\pi M_{\perp} \quad (7-30)$$

where  $H$  is the magnetic field given by the solution of the Maxwell equations for the conductor. In a direction parallel to the filaments, the local field is the same as  $H$ , but in the transverse direction the local field differs from  $H_{\perp}$  unless it is large enough for  $2\pi M_{\perp}$  to be neglected. The magnetization saturates when the filaments are fully penetrated, and it is easily shown from (7-16) that the saturation value is (for no transport current and only bulk screening currents considered)

$$M_{sat \perp} = -\frac{\lambda}{6\pi} H_p \quad (7-31)$$

---

9. The expression is analogous to the local field  $H_{loc} = H + \frac{4\pi}{3} M$  in a ferromagnet resulting from the removal of an atom.  $4\pi/3$  is the demagnetizing factor of a sphere whereas  $2\pi$  is the demagnetizing factor of a cylinder.

where  $H_p$  is the transverse penetration field for a filament of diameter  $d$ , with surface current neglected, given by

$$H_p = 4 d j_c . \quad (7-32)$$

The magnetization due to the surface current should be added to (7-31) and consequently, in total,  $2\pi M_{sat}$  is typically in a range from  $H_{c1}$  to several thousand oersteds. For weak  $H$  fields of this order of magnitude or less, the distinction between  $H$  and the local field cannot be ignored. The field  $H$  is the sum of the field  $H_A$  applied to the conductor and the demagnetizing field due to  $M$  in the conductor, plus the field due to internal currents. One may estimate the local field under some conditions of interest as follows:

(a) A Slowly Changing Applied Field and No Transport Current

For a slowly changing or low frequency applied field, shielding effects may be ignored, and

$$\vec{H} = \vec{H}_A - N_x M_x \hat{a}_x - N_y M_y \hat{a}_y \quad (7-33)$$

where  $N_x$  and  $N_y$  are demagnetizing factors for the conductor, and  $\hat{a}_x$  and  $\hat{a}_y$  are unit vectors along the  $x$  and  $y$  axes, with the  $z$  direction along the axis of the conductor. For a flat conductor  $N_x$  and  $N_y$  may be taken to be the values for a long ellipsoid of the same aspect ratio. For a circular conductor  $N_x = N_y = 2\pi$ , and if a long twist length is assumed,

so that a direction transverse to the conductor is approximately transverse to the filaments,

$$\underline{H} \approx \underline{H}_A - 2\pi \underline{M}_\perp \quad (7-34)$$

Therefore from (7-30)

$$H_{loc} = H_A \quad (7-35)$$

for all field strengths.

#### (b) Rapidly Changing Fields

For rapidly changing transverse fields where induced currents shield the interior of the conductor,  $\underline{H}$  is reduced to a small value,<sup>3</sup> if the twist length is sufficiently long. For strong shielding the local field in the interior may be set equal to zero.

#### 7.7 Values for the Permeability

The magnetic moment of a filament is a function of the local field acting on the filament. From the expression for  $\underline{M}_\perp$  given by (7-16) it follows for circular filaments of diameter  $d$  that

$$M_\perp = \frac{4\lambda}{\pi d^2} \int (y \hat{a}_x - x \hat{a}_y) \cdot \underline{H} \, da \quad (7-36)$$

where the coordinates should refer to the filament, with  $z$  the filament axis, but for typical twist lengths they differ only slightly from the conductor coordinates. Consider a given filament having no transport current but subjected to an applied local field in the  $x$  direction. Then  $i_{||}$  will be an even function of  $x$  and an odd function of  $y$ , and consequently

$$M_{\perp} = \hat{a}_x \frac{4\lambda}{\pi d^2} \int y i_{||} da. \quad (7-37)$$

The interest here is in the dynamic permeability determined by the change in magnetization due to a change in field (Figure 7-4). From the current distribution given in Section 6.2 for weak partial penetration, it follows that for a transverse local field

$$\frac{\Delta M_{\perp}}{\Delta H_{loc}} = \frac{-\lambda}{2\pi}. \quad (7-38)$$

For the opposite case of a strong local field  $\Delta M_{\perp}/\Delta H_{loc}$  approaches zero. The results may be expressed in terms of the Maxwell field  $H$  by the use of Eq. (7-30). If the dynamic permeability is defined by

$$\mu = 1 + 4\pi \frac{\Delta M}{\Delta H} \quad (7-39)$$

then it is easily shown that for a transverse local field

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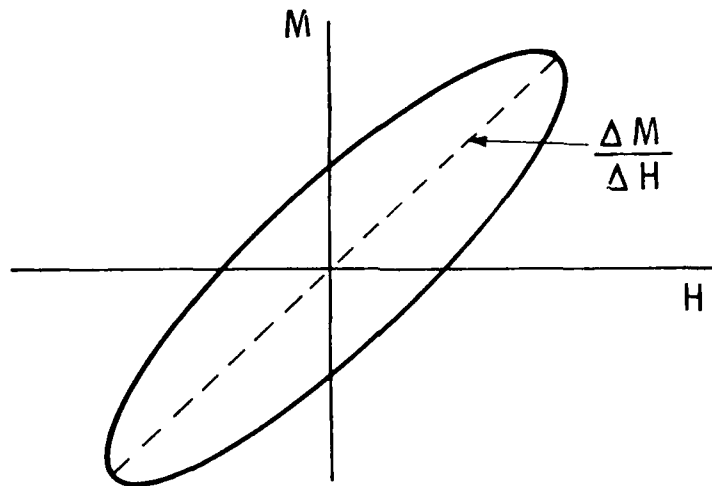


Fig. 7. 4 — The dynamic susceptibility

$$\mu_{\perp} = \frac{1 - \lambda}{1 + \lambda} \quad (H_{loc\ o} < < H_p) \quad (7-40)$$

$$\mu_{\perp} = 1 \quad (H_{loc\ o} > > H_p) \quad (7-41)$$

where  $H_{loc\ o}$  indicates the amplitude of a cyclic local field and  $H_p$  is the transverse penetration field for a filament. In a similar way, for a local field along the filament axis, where  $H_{loc} = H$ , it can be shown that

$$\mu_{||} = 1 - \lambda \quad (H_o < < H_p) \quad (7-42)$$

$$\mu_{||} = 1 \quad (H_o > > H_p) \quad (7-43)$$

where  $H_p$  is the longitudinal penetration field, equal to  $2\pi d j_c$  neglecting surface current. As in a ferromagnet, the values for the permeabilities do not necessarily follow a principal of superposition. If one alternating field component is large compared with  $H_p$  then both permeabilities may be set equal to unity. Transport current also affects the permeability, tending to increase the value towards unity. For a dc transport current equal to the critical current of the filament, no magnetization exists and the permeability is equal to one.

### 7.8 Conductivity for a Single Component Matrix

The most important material constant is the transverse conductivity  $\sigma_{\perp}$ , which represents the ratio of the average transverse current density to the average electric field transverse to the filament axis.



Values for  $\sigma_{\perp}$  have been calculated for the two cases illustrated in Figure 7-5. In one case a large resistance exists at the filament-matrix interface, and the eddy currents tend to flow around the filament, while in the second case this resistance is absent and current flows through the filaments. If no interfacial resistance exists, then to good approximation<sup>7</sup>

$$\sigma_{\perp} = \sigma_m \frac{(1 + \lambda)}{1 - \lambda} \quad (7-44)$$

where  $\sigma_m$  is the conductivity of the matrix. It is assumed in this expression that the filament is not saturated with transport current, and that relative to the matrix the conductivity of the superconducting filament is infinite. Therefore when the fractional amount of superconductor  $\lambda$  approaches unity  $\sigma_{\perp}$  approaches infinity. The expression is relatively independent of geometry, if the geometry is reasonably simple, but obviously the values are least accurate as  $\lambda \rightarrow 1$ , since due to irregularities, continuous paths through the filaments are possible even when  $\lambda$  is less than unity.

For the case where a large resistance exists at the interface between filament and matrix, an approximation for the conductivity is given by

$$\sigma_{\perp} = \sigma_m \frac{(1 - \lambda)}{1 + \lambda} \quad (7-45)$$

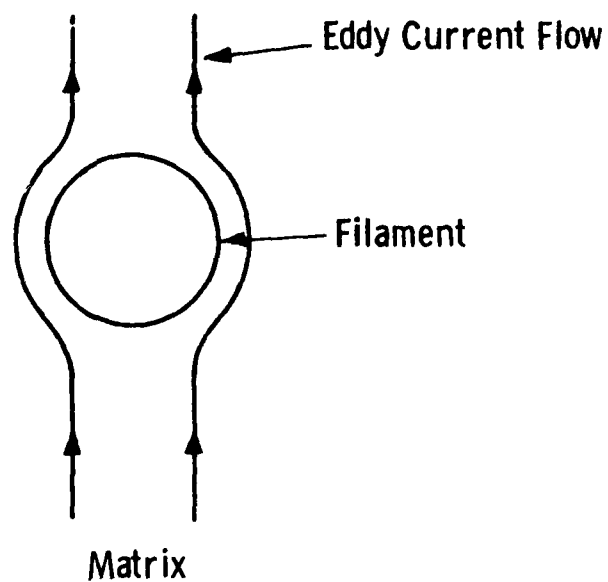
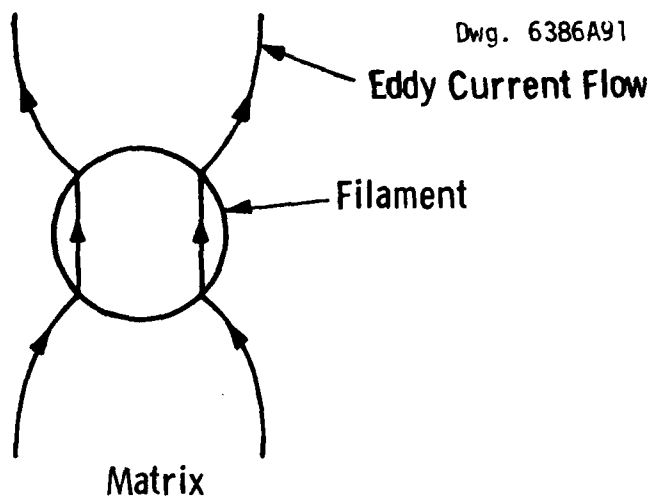


Fig. 7. 5 — Eddy current flow in the neighborhood of a filament with ( a ) no interfacial resistance and ( b ) high resistance.

which goes to zero as  $\lambda \rightarrow 1$ . The reason this expression is of some interest is that during a heat treatment in the manufacturing process, diffusion can occur between filament and matrix forming an alloy layer around the filament,<sup>10</sup> and also contact resistance can occur due to defects.<sup>11</sup> If the matrix itself is an alloy with relatively high resistivity, the layer is unimportant, but for a pure copper matrix it becomes important. In qualitative agreement with this assertion, it is usually found experimentally that the expression (7-44) tends to apply for the case of an alloy matrix such as copper-nickel, while (7-45) is a better approximation for a pure copper matrix. The results of an experiment to investigate the validity of (7-45) are shown in Figure 7-6.<sup>12</sup> Measurements of  $\sigma_{\perp}$  in actual conductors have been made by Davoust and Renard.<sup>11</sup>

Due to magnetoresistance  $\sigma_m$  is a function of the magnetic field. Also, Walker<sup>13</sup> has pointed out that for fine filaments in pure Cu the spacing between filaments can be less than the electron mean free path, and  $\sigma_m$  is no longer given by the value for bulk material.

#### 7.9 Expressions for the Loss in a Composite Conductor

The power loss for the composite in terms of its macroscopic fields is

- 
10. M. Wake, K. Ishibashi, M. Kobayashi and B. Turck, Sixth International Conference on Magnet Tech., 734, Pub. by Alfa, Bratislava, Czech.
  11. M. E. Davoust and J. C. Renard, Proc. Sixth International Cryogenic Eng. Conf., 458, Ed. K. Mendelssohn (1976).
  12. Demonstration of an Advanced Superconducting Generator, Interim Rpt., 1-31-77, Cry. Eng. Lab. and Elec. Pwr. Sys. Eng. Lab., School of Engineering, Massachusetts Institute of Technology.
  13. M. S. Walker, J. H. Murphy and W. J. Carr, Jr., IEEE Trans. on Mag., MAG-11, 309 (1975).

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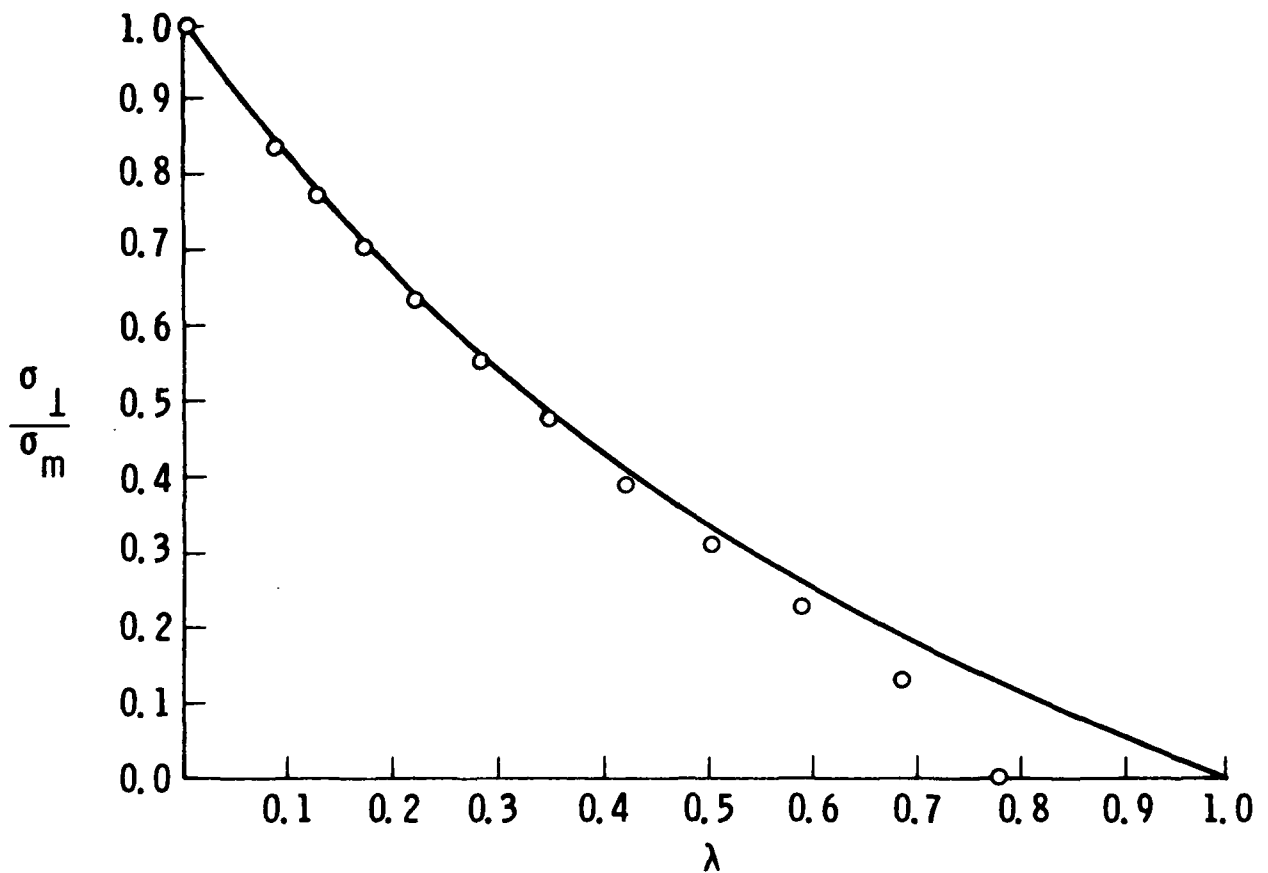


Fig. 7. 6 — Plot of  $\sigma_{\perp} / \sigma_m$  vs  $\lambda$ . ——— given by Eq. (7-45). Data points taken from measurements made on a conducting paper with circular holes. <sup>10</sup>

$$\oint P \, dt = \oint dt \int_V \underline{j} \cdot \underline{E} \, dV + \int_V dV \oint \underline{H} \cdot d\underline{M} . \quad (7-46)$$

For the constitutive Eq. (7-26) the eddy current part of (7-46) is

$$\oint P_e \, dt = \oint dt \int_V \frac{j_{\perp}^2}{\sigma_{\perp}} \, dV \quad (7-47)$$

but the hysteresis part is considerably more difficult to evaluate. Unlike the case of a pure bulk superconductor, the magnetization cannot be neglected, and a direct evaluation of the hysteresis requires a knowledge of the phase relation between  $\underline{M}$  and  $\underline{H}$ . However, such evaluation is avoided by the approach outlined in 7.2, which requires only the magnitude of a local field.<sup>14</sup> Let the composite be divided into two regions, one having current saturated filaments, the other current unsaturated. In the latter, on a microscopic picture the electric field  $\underline{e}$  will vanish on some surface within each filament, and each filament may be treated as a separate bulk superconductor. For a single bulk superconductor the hysteresis loss per unit volume depends upon its applied field and the current flowing in the superconductor, given in functional form by

$$\oint \frac{P_h}{V} \, dt = F \left( \frac{H}{H_A}, \frac{I}{I_c} \right) . \quad (7-48)$$

14. Irie et al. have taken a different approach and introduced a complex permeability: F. Irie, F. Sumiyoshi and K. Yoshida, IEEE Trans. on Magnetics, MAG-15, 244 (1979). In this approach the continuum picture is used to calculate the hysteresis as well as the eddy current loss, while the method described in the text makes use of a discrete filament model for the hysteresis, in the unsaturated region.

If each section of filament is treated as such a conductor its loss per unit volume is then given approximately by  $F(H_{loc}, j_{||}/\lambda j_c)$ . The loss per unit volume of the composite at any point is  $\lambda F$  and for the unsaturated region of the composite conductor

$$\oint P_h dt = \lambda \int_{V_{unsat.}} dV F\left(H_{loc}, \frac{j_{||}}{\lambda j_c}\right) \quad (7-49)$$

The total loss in this region is the sum of (7-49) and (7-47). Either the London-Bean or some more exact approximation may be used for  $F$ . The unsaturated region is defined by  $j_{||}/\lambda j_c \leq 1$ , while in the saturated region  $j_{||}/\lambda j_c$  is slightly greater than unity. The saturated region tends to be near the surface of the composite conductor and results, for example, from eddy currents flowing into the surface or from alternating transport current in the conductor. The electric field in this region is determined by the thickness of the saturated layer, and not by the filament geometry. If the twist length is reasonably long so that the saturated current flows nearly along the axis of the conductor, the electric field is nearly the same as that for a pure bulk superconductor of the same dimensions as the composite, with critical current density  $\lambda j_c$ , which is penetrated corresponding to the depth of the saturated layer. Consequently the loss in the saturated layer of a composite wire may be estimated from the loss of a solid superconductor of the same diameter. The eddy current loss in the saturated region may also be computed, including a part due to  $E_{||}$ , but this is usually negligible

compared with the hysteresis part and will be ignored. The whole layer may be ignored if its thickness is very small compared with the conductor radius, and in particular if it is less than the first layer of filaments.

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